

Scenario for Ultrarelativistic Nuclear Collisions:

IV. Effective quark mass at the early stage.

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Using the framework of wedge dynamics, we compute the effective transverse mass of a soft quark mode propagating in the expanding background of hard quarks and gluons created at the earliest time of the collision. We discover that the wedge dynamics does not require any external infrared or collinear cut-off. The effective mass is produced mainly due to the forward quark-quark scattering mediated by the longitudinal (in sense of Gauss' law) magnetic fields. Contribution of the radiation field is parametrically suppressed.

I. INTRODUCTION

In the first paper of this cycle [1] (further quoted as paper [I]), we formulated a program that might result in a theory of ultrarelativistic nuclear collisions which is free from collinear problems and naturally establishes the infrared boundary for the space of “final” states at the very early stage of the collision ($\leq 1fm$). We have demonstrated that even at very early times (much less than is required for any kinetic process to develop), the collective interactions in a dense system provide the final states of the QCD evolution with finite dynamically generated masses that shield mass singularities in the evolution equations.¹ It was shown also that the null-plane dynamics are incapable of describing local screening effects, because any type of kinetics is frozen on the light cone. It was suggested, that a more adequate approach requires the change of the global Hamiltonian dynamics which is used for the field-theory description of nuclear collisions. We proposed the so-called *wedge dynamics* which employs the proper time τ measured from the first touch of the Lorenz-contracted nuclei as the Hamiltonian time. Our initial estimates in paper [I] were very qualitative. In two consecutive papers [4,5] (further quoted as papers [II] and [III]), we have studied, in detail, the space of states of wedge dynamics. In paper [II], we extended the qualitative analysis of scalar fields initiated in [I], and found that for the charged fields, the early-time evolution of the wave function is accompanied by a gradual rearrangement of the charge distribution, starting from its almost uniform spread along the light cone at $\tau \rightarrow 0$, and up to a narrow wave packet with a well defined rapidity at later times. We have shown that this re-distribution of the charge leads to currents in the rapidity direction and that these currents are the largest at the earliest τ . The magnetic fields generated by these currents can be responsible for the interactions between the currents at the earliest moments of the QCD evolution. In paper [II], we studied the states of fermions in wedge dynamics and found the fermion field correlators that are used below for the calculation of the quark self-energy in the expanding system. In paper [III], we addressed the issue of gauge fields in wedge dynamics. Several important problems were solved there. The natural gauge condition of wedge dynamics, $A^\tau = 0$, was proved to be completely fixed (at the level of perturbation theory). The second (technically nearly most difficult) problem solved in paper [III], was the separation of the longitudinal (i.e., governed by Gauss' law) field and the field of radiation. In that paper, we also quantized the gauge field in the scope of wedge dynamics and explicitly found the Wightman functions and retarded propagator of the gluon field which are used in this paper for the practical calculation of the fermion self-energy.

¹The idea that screening effects should be taken into account at the early kinetic stage of a collision has been articulated earlier and with different motivations by Shuryak and Xiong [2] and by Eskola, Muller and X.-N. Wang [3]

Our decision to begin the exploration of potentialities of the wedge dynamics with the computation of quark self-energy is motivated only by technical reasons. The gluon propagator of wedge dynamics is a very complicated function, and we preferred to start with the computation of the fermion loop which has only one gluon correlator in it. We hope that the possibility of a technical simplification (compared to what we had to start with) discovered in this paper, will allow us to address a more important problem of the gluon self-energy in a reasonably economic way.

In the course of this study, we employ a single heuristic assumption (supported by the analysis of paper [II]) that the field states with large transverse momentum, even at very early times, may be associated with the localized particles and thus can be described by the distribution with respect to the rapidity and transverse momentum. Our strategy of looking for the leading contributions, as well as all our approximations, in the calculation of the material part of the quark self-energy are based on this assumption. If it appears incorrect, then it is most likely that the quark-gluon matter created in the collision of two nuclei never and in no approximation can be considered as a system of nearly free and weakly interacting field states.

II. FERMION RETARDED SELF-ENERGY

In order to find the normal modes of the quark field in the expanding quark-gluon system, we are going to solve the Dirac equation with the radiative corrections, which can be derived as a projection of the Schwinger-Dyson equation for the retarded quark propagator onto the one-particle initial state. For the quark field without Lagrangian mass, this equation reads

$$i\gamma^\mu(x_1)\nabla_\mu(x_1)\psi(x_1) = \int d^4x_2 \Sigma_{[ret]}(x_1, x_2)\psi(x_2) . \quad (2.1)$$

The covariant derivative $\nabla_\mu(x)$ of the spinor field in the curvilinear coordinates of the wedge dynamics includes the spin connection and it was found explicitly in paper [II]. For all calculations below, we employ the mixed representation which is the most profitable in heavy-ion problems. We are looking for the radiative corrections to the wave function with a given transverse momentum \vec{p}_t and rapidity θ with the expectation that within the rapidity plateau nothing will depend on θ . However, we cannot totally eliminate the coordinate η from the theory. We have to keep it explicitly, since the problem of the expanding field system cannot be reduced to (2+1) dimensions. In its expanded form, Eq. (2.1) reads,

$$\left[i\gamma^0\left(\frac{\partial}{\partial\tau_1} + \frac{1}{2\tau_1}\right) + \frac{i\gamma^3}{\tau_1}\frac{\partial}{\partial\eta_1} - p_r\gamma^r \right] \psi(p_t; \tau_1, \eta_1) = \int_0^{\tau_1} d\tau_2 \int_{-\infty}^{\infty} \tau_2 d\eta_2 \Sigma_{[ret]}(p_t; \tau_1, \tau_2; \eta_1 - \eta_2) \psi(p_t; \tau_2, \eta_2) . \quad (2.2)$$

The retarded self-energy is an object that naturally emerges in the Schwinger-Dyson equation for the retarded propagator in Keldysh-Schwinger formalism [6]. Below, we employ its modified form developed earlier with the view of application to the inclusive and transient processes. We employ the notation used in Refs. [7,8,1].² In this notation, the one-loop retarded fermion self-energy in coordinate form is

$$\Sigma_{[ret]}(x_1, x_2) = \frac{ig^2}{2} [t^a \gamma^\mu G_{[ret]}(x_1, x_2) t^b \gamma^\lambda D_{[1]\lambda\mu}^{ba}(x_2, x_1) + t^a \gamma^\mu G_{[1]}(x_1, x_2) t^b \gamma^\lambda D_{[adv]\lambda\mu}^{ba}(x_2, x_1)] . \quad (2.3)$$

The two subprocesses that contribute this self-energy are depicted below.

²The indices of the field correlators with the Keldysh contour ordering of the field operators (like $G_{[AB]}$) as well as the labels of their linear combinations (like $G_{[ret]}$) are placed in square brackets.

FIG. 1. The retarded forward scattering amplitude is contributed by two subprocesses, $qg \rightarrow qg$ and $qq \rightarrow qq$.

The retarded and advanced quark and gluon propagators $G_{[ret]}$ and $D_{[adv]}^{lm}$ were found in papers [II] and [III] of this cycle and are connected with the commutators $G_{[0]}$ and $D_{[0]}$,

$$G_{[ret]}(x_1, x_2) = \theta(\tau_1 - \tau_2)G_{[0]}(x_1, x_2), \quad D_{[adv]}^{lm}(x_2, x_1) = -\theta(\tau_1 - \tau_2)D_{[0]}^{lm}(x_2, x_1) + D_L^{lm}(x_2, x_1), \quad (2.4)$$

where $D_L^{lm}(x_2, x_1)$ is the longitudinal part of the gluon propagator (governed by Gauss' law), and it enters in Eq. (2.4) in such a way that the condition $D_{[ret]} - D_{[adv]} = D_{[0]}$ is satisfied and the non-causal longitudinal part of the propagator does not violate the causal properties of the commutator $D_{[0]}$. The correlators $G_{[1]}$ and $D_{[1]}$ include densities of vacuum states as well as the information about the occupation numbers (phase-space population). Eventually, we shall prove that an approximation of the boost-invariance (infinite rapidity plateau) is not corrupted by any kind of cut-offs (the vacuum part never is). Therefore, all correlators (G , D , and Σ) will depend on two times τ_1 and τ_2 separately, the difference of rapidities $\eta = \eta_1 - \eta_2$, and the difference $\vec{r} = \vec{r}_1 - \vec{r}_2$ of distances in xy plane. The latter is Fourier transformed to the transverse momentum dependence. In this mixed representation,

$$\begin{aligned} \Sigma_{[ret]}(\tau_1, \tau_2; \eta, \vec{p}_t) = & \frac{ig^2}{2(2\pi)^2} \int d^2\vec{k}_t [t^a \gamma^m(\tau_1) G_{[ret]}(\tau_1, \tau_2; \eta, \vec{p}_t + \vec{k}_t) t^b \gamma^l(\tau_2) D_{[1]lm}^{ba}(\tau_2, \tau_1; -\eta, \vec{k}_t) \\ & + t^a \gamma^m(\tau_1) G_{[1]}(\tau_1, \tau_2; \eta, \vec{p}_t + \vec{k}_t) t^b \gamma^l(\tau_2) D_{[adv]lm}^{ba}(\tau_2, \tau_1; -\eta, \vec{k}_t)], \end{aligned} \quad (2.5)$$

where $\gamma^\eta(\tau) = \gamma^3/\tau$. As it has been shown in [I], all fermion correlators $G_{[\alpha]}$ can be decomposed as

$$\begin{aligned} G_{[\alpha]}(\tau_1, \tau_2; \eta, \vec{q}_t) = & q_t [g_{[\alpha]}^0 \gamma^0 + g_{[\alpha]}^3 \gamma^3] + g_{[\alpha]}^T q_r \gamma^r + i g_{[\alpha]}^A q_r \epsilon^{ru} \gamma^u \gamma^5 \\ = & q_t [g_{[\alpha]}^{L(+)} \gamma^+ + g_{[\alpha]}^{L(-)} \gamma^-] + q_r \gamma^r \gamma^0 [g_{[\alpha]}^{T(+)} \gamma^+ + g_{[\alpha]}^{T(-)} \gamma^-], \end{aligned} \quad (2.6)$$

where, for the sake of brevity, we denote $\vec{q}_t = \vec{p}_t + \vec{k}_t$. A similar decomposition takes place for the self-energy,

$$\begin{aligned} \Sigma_{[ret]}(\tau_1, \tau_2; \eta, \vec{p}_t) = & \Sigma^0 \gamma^0 + \Sigma^3 \gamma^3 + \Sigma^T q_r \gamma^r + i \Sigma^A p_r \epsilon^{ru} \gamma^u \gamma^5 \\ = & \Sigma^{L(+)} \gamma^+ + \Sigma^{L(-)} \gamma^- + p_r \gamma^r \gamma^0 [\Sigma^{T(+)} \gamma^+ + \Sigma^{T(-)} \gamma^-], \end{aligned} \quad (2.7)$$

and we obviously have,

$$g_{[\alpha]}^{L(\pm)} = \frac{1}{2}(g_{[\alpha]}^0 \pm g_{[\alpha]}^3), \quad g_{[\alpha]}^{T(\pm)} = \frac{1}{2}(g_{[\alpha]}^T \pm g_{[\alpha]}^A), \quad \Sigma^{L(\pm)} = \frac{1}{2}(\Sigma^0 \pm \Sigma^3), \quad \Sigma^{T(\pm)} = \frac{1}{2}(\Sigma^T \pm \Sigma^A). \quad (2.8)$$

It becomes easier to analyze the various pieces of the quark self-energy if the gluon correlators $D_{[\alpha]lm}$ are taken in the form of the following decomposition,³

$$D_{[\alpha]rs} = \left(\delta_{rs} - \frac{k_r k_s}{k_t^2} \right) \mathcal{D}_{[\alpha]}^{(TE)} + \frac{k_r k_s}{k_t^2} \mathcal{D}_{[\alpha]}^{(2)}, \quad D_{[\alpha]\eta\eta} = \mathcal{D}_{[\alpha]}^{(\eta\eta)}, \quad D_{[\alpha]r\eta} = \frac{k_r}{k_t^2} \mathcal{D}_{[\alpha]}^{(r\eta)}, \quad D_{[\alpha]\eta s} = \frac{k_s}{k_t^2} \mathcal{D}_{[\alpha]}^{(\eta s)}, \quad (2.9)$$

where the first term in $D_{[\alpha]rs}$ is due to the transverse electric mode, and all invariants of $\mathcal{D}_{[adv]}$ (except $\mathcal{D}_{[adv]}^{(TE)}$) have two terms, $\mathcal{D}_{[0]}^{(\cdots)}$ from the transverse magnetic mode of the radiation field, and $\mathcal{D}_{[long]}^{(\cdots)}$ from the longitudinal field. All these components were found in paper [III] and are given in the Appendix in the form which is used in the calculation below. After some algebra, we can present the retarded quark self-energy in the form,

$$\Sigma_{[ret]}(\tau_1, \tau_2; \eta, \vec{p}_t) = \frac{i\alpha_s C_F}{2\pi} \int d^2 \vec{k}_t [\gamma^+ S^{L(+)} + \gamma^- S^{L(-)} + p_r \gamma^r \gamma^0 (\gamma^+ S^{T(+)} + \gamma^- S^{T(-)})], \quad (2.10)$$

where the scalar invariants of $\Sigma_{[ret]}$ are the bilinears of the fermion and gluon scalars,

$$S^{L(\pm)} = \sum_{[\alpha, \beta]} \left\{ q_t g_{[\alpha]}^{L(\pm)} (\mathcal{D}_{[\beta]}^{(TE)} + \mathcal{D}_{[\beta]}^{(2)}) + \frac{q_t}{\tau_1 \tau_2} g_{[\alpha]}^{L(\mp)} \mathcal{D}_{[\beta]}^{(\eta\eta)} \mp \frac{(\vec{k}_t \vec{q}_t)}{k_t^2} \left(\frac{g_{[\alpha]}^{T(\pm)} \mathcal{D}_{[\beta]}^{(r\eta)}}{\tau_1} + \frac{g_{[\alpha]}^{T(\mp)} \mathcal{D}_{[\beta]}^{(\eta r)}}{\tau_2} \right) \right\}, \quad (2.11)$$

$$S^{T(\pm)} = \sum_{[\alpha, \beta]} \left\{ \left[\frac{(\vec{p}_t \vec{q}_t)}{p_t^2} - 2 \frac{(\vec{k}_t \vec{p}_t)(\vec{k}_t \vec{q}_t)}{k_t^2 p_t^2} \right] g_{[\alpha]}^{T(\pm)} (\mathcal{D}_{[\beta]}^{(TE)} + \mathcal{D}_{[\beta]}^{(2)}) - \frac{(\vec{q}_t \vec{p}_t)}{p_t^2 \tau_1 \tau_2} g_{[\alpha]}^{T(\mp)} \mathcal{D}_{[\beta]}^{(\eta\eta)} \mp \frac{(\vec{k}_t \vec{p}_t)}{k_t^2 p_t^2} \left(\frac{g_{[\alpha]}^{L(\pm)} \mathcal{D}_{[\beta]}^{(r\eta)}}{\tau_1} - \frac{g_{[\alpha]}^{L(\mp)} \mathcal{D}_{[\beta]}^{(\eta r)}}{\tau_2} \right) \right\}. \quad (2.12)$$

In these equations, the sum $\sum_{[\alpha, \beta]}$ runs over $[\alpha, \beta] = \{[ret, 1], [1, adv]\}$.

III. FERMION MODES IN THE EXPANDING SYSTEM

We shall look for the dispersion law of the fermions in the proper-time dynamics studying the Dirac equation (2.2) with radiative corrections. Since the fermions are massless, it is convenient to use the spinor basis where the Dirac matrices are

$$\gamma^0 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}, \quad \gamma^l = \begin{pmatrix} 0 & -\sigma^l \\ \sigma^l & 0 \end{pmatrix},$$

and the Dirac equation can be split into two separate equations for the left- and right-handed two-component spinors. The latter reads as

$$G_R^{-1}(p_t; \tau_1, \eta_1) \psi_R(p_t; \tau_1, \eta_1) = \int_0^{\tau_1} d\tau_2 \int_{-\infty}^{\infty} \tau_2 d\eta_2 \Sigma_R(p_t; \tau_1, \tau_2; \eta_1 - \eta_2) \psi_R(p_t; \tau_2, \eta_2), \quad (3.1)$$

where the matrices of the right-handed differential operator G_R^{-1} and of the right-handed self-energy Σ_R are

³In what follows, we use the Greek indices for the four-dimensional vectors and tensors in the curvilinear coordinates (index η is an exception, it always denotes the rapidity direction), and the Latin indices from a to d for the vectors in flat Minkowski coordinates. We use Latin indices from r to w for the transverse x - and y -components ($r, \dots, w = 1, 2$), and the arrows over the letters to denote the two-dimensional vectors, *e.g.*, $\vec{k} = (k_x, k_y)$, $|\vec{k}| = k_t$. The Latin indices from i to n ($i, \dots, n = 1, 2, 3$) will be used for the three-dimensional internal coordinates $u^i = (x, y, \eta)$ on the hyper-surface $\tau = const$.

$$G_R^{-1}(p_t; \tau, \eta) \begin{bmatrix} i(\partial_\tau + \frac{1}{2\tau} - \frac{1}{\tau}\partial_\eta) & p_x - ip_y \\ p_x + ip_y & i(\partial_\tau + \frac{1}{2\tau} + \frac{1}{\tau}\partial_\eta) \end{bmatrix},$$

$$\Sigma_R(p_t; \tau_1, \tau_2; \eta_1 - \eta_2) = \begin{bmatrix} \Sigma^{L(-)} & -(p_x - ip_y)\Sigma^{T(+)} \\ -(p_x + ip_y)\Sigma^{T(-)} & \Sigma^{L(+)} \end{bmatrix}. \quad (3.2)$$

The equation for the left-handed spinors differs from (3.1) only by a change of some signs in matrices (3.2) and leads to the same dispersion law. A solution with positive energy is looked for in the form,

$$\psi_R(p_t, \theta; \tau, \eta) = \begin{pmatrix} e^{(\eta-\theta)/2} p_t \\ -e^{-(\eta-\theta)/2} (p_x + ip_y) \end{pmatrix} e^{-i\mu\tau \cosh(\eta-\theta)}, \quad (3.3)$$

where μ is the effective “transverse mass” of the mode. For the free on-mass-shell solution we have $\mu = p_t$. To solve Eq. (3.1), we introduce an auxiliary (left-handed) spinor

$$\tilde{\psi}(p_t, \theta'; \tau, \eta) = \begin{pmatrix} e^{-(\eta-\theta')/2} p_t \\ -e^{(\eta-\theta')/2} (p_x - ip_y) \end{pmatrix} e^{i\mu\tau \cosh(\eta-\theta')}. \quad (3.4)$$

We insert (3.3) into (3.1), multiply it from the left by (3.4) and integrate this along the hypersurface $\tau_1 = \text{const.}$ Then the left side of the equation becomes

$$\int_{-\infty}^{\infty} \tau_1 d\eta_1 \tilde{\psi}(p_t, \theta'; \tau_1, \eta_1) G_R^{-1}(p_t; \tau_1, \eta_1) \psi_R(p_t, \theta; \tau_1, \eta_1) = 4\pi \frac{\mu - p_t}{\mu} p_t^2 \delta(\theta - \theta'). \quad (3.5)$$

In deriving this equation, we assumed that μ is independent of τ_1 . The weak dependence is admissible, provided $d\mu/d\tau_1 \ll \mu/\tau_1$. A solution that has this property does indeed exist. The right hand side of the equation is, in fact, independent of θ' and is of the following form,

$$p_t^2 \int_0^{\tau_1} d\tau_2 \int_{-\infty}^{\infty} \tau_1 \tau_2 d\eta_2 d\theta d(\eta_1 - \eta_2) e^{i\mu\tau_1 \cosh(\eta_1+\theta)} e^{-i\mu\tau_2 \cosh \eta_2} \\ \times [e^{-\frac{\eta_1-\eta_2+\theta}{2}} \Sigma^{(L)-} + e^{\frac{\eta_1-\eta_2+\theta}{2}} \Sigma^{(L)+} + e^{-\frac{\eta_1+\eta_2+\theta}{2}} \Sigma^{(T)+} + e^{\frac{\eta_1+\eta_2+\theta}{2}} \Sigma^{(T)-}] , \quad (3.6)$$

where the exponentials are due to the Thomas precession of the spinor field. Next, we integrate both sides with respect to θ . Two rapidity integrals, $d\theta d\eta_2$, on the right absorb the precession factors yielding the product of Hankel functions,

$$\pi^2 H_{1/2}^{(1)}(\mu\tau_1) H_{1/2}^{(2)}(\mu\tau_2) = \frac{2\pi}{\mu\sqrt{\tau_1\tau_2}} e^{i\mu(\tau_1-\tau_2)}. \quad (3.7)$$

Finally, we arrive at the dispersion equation that defines the fermion “transverse mass” μ as a function of the transverse momentum and the latest time τ_1 ,

$$\mu(p_t, \tau_1) - p_t = \frac{1}{2} \int_0^{\tau_1} d\tau_2 \sqrt{\tau_1\tau_2} e^{i\mu(p_t, \tau_1)(\tau_1-\tau_2)} \int_{-\infty}^{\infty} d\eta [\Sigma^{L(+)} + \Sigma^{L(-)} + p_t \Sigma^{T(+)} + p_t \Sigma^{T(-)}]. \quad (3.8)$$

As it has been discussed in paper [II] for fermions (similar arguments are true for gluons), only the independence of the quark and gluon occupation numbers n_f and n_g on rapidity can provide that the invariants $S^{L(\pm)}$ and $S^{T(\pm)}$ naturally depend only on the difference $\eta = \eta_1 - \eta_2$. We shall consider only this case of the local homogeneity; we can do it safely only because no collinear singularities which may require a rapidity cut-off (e.g., $\pm Y$) in the phase space will appear in the theory. Since we are computing an essentially local quantity, such a cut-off would be unphysical. With this reservation, we may rewrite Eq. (3.8) as

$$\mu(p_t) = p_t + \int_0^{\tau_1} d\tau_2 \sqrt{\tau_1\tau_2} e^{i\mu(p_t)(\tau_1-\tau_2)} [\Sigma^0(\tau_1, \tau_2) + p_t \Sigma^T(\tau_1, \tau_2)], \quad (3.9)$$

where we introduced the notation,

$$\Sigma^0(\tau_1, \tau_2) = \frac{i\alpha_s C_F}{4\pi} \sum_{[\alpha, \beta]} \int d^2 \vec{k}_t \int_{-\infty}^{\infty} d\eta \, q_t g_{[\alpha]}^0 [\mathcal{D}_{[\beta]}^{(TE)} + \mathcal{D}_{[\beta]}^{(2)} + \frac{1}{\tau_1 \tau_2} \mathcal{D}_{[\beta]}^{(\eta\eta)}], \quad (3.10)$$

$$\Sigma^T(\tau_1, \tau_2) = \frac{i\alpha_s C_F}{4\pi} \sum_{[\alpha, \beta]} \int d^2 \vec{k}_t \int_{-\infty}^{\infty} d\eta \, g_{[\alpha]}^T \left\{ \left[\frac{(\vec{p}_t \vec{q}_t)}{p_t^2} - 2 \frac{(\vec{k}_t \vec{p}_t)(\vec{k}_t \vec{q}_t)}{k_t^2 p_t^2} \right] (\mathcal{D}_{[\beta]}^{(TE)} + \mathcal{D}_{[\beta]}^{(2)}) + \frac{(\vec{q}_t \vec{p}_t)}{p_t^2 \tau_1 \tau_2} \mathcal{D}_{[\beta]}^{(\eta\eta)} \right\}. \quad (3.11)$$

Comparing these equations with Eqs. (2.11) and (2.12), we may observe a significant simplification. The terms with the off-diagonal components $\mathcal{D}^{(\eta r)}$ and $\mathcal{D}^{(r\eta)}$ have dropped out. These terms, as it can be seen from Eqs. (A6)–(A7), (A10)–(A11), and (A18)–(A19), are odd with respect to η , while the invariants $g^0 = g^{L(+)} + g^{L(-)}$ and $g^T = g^{T(+)} + g^{T(-)}$ are even. Therefore, integration over η eliminates the terms with the off-diagonal components.

IV. PROPAGATORS, DENSITIES OF STATES, AND OCCUPATION NUMBERS IN THE EXPANDING SYSTEM

In this section, we collect condensed information about various correlators of quark and gluon fields derived in papers [II] and [III] which are necessary for the calculation of the quark self-energy. We also discuss our specific choice of occupation numbers $n_g(k_t, \alpha)$ and $n_f(q_t, \theta)$. All field correlators are defined as the expectation values over the distribution of the background particles. The latter are the excitations of the modes allowed by the constraints and the boundary conditions of wedge dynamics. The Fock space of these excitations was constructed in papers [II] and [III]. We have analyzed two sets of quantum numbers that may label the states. Both sets include the transverse momentum \vec{p}_t and polarization index. In one set, the remaining variable was the boost ν (the variable conjugated to the coordinate η); this set proved to be very useful in the practical calculation of the gluon propagators. In the second set, the particles are labeled by their velocity $v_z = \tanh \theta$ in the direction of the collision axis. This representation is used below. The fermion spectral functions are

$$\begin{aligned} G_{[10]}(q_t, \theta; \tau_1, \tau_2) &= [1 - n_f^+(q_t, \theta)] G_{[10]}^{(0)}(q_t, \theta; \tau_1, \tau_2) - n_f^-(q_t, \theta) G_{[01]}^{(0)}(q_t, \theta; \tau_1, \tau_2) , \\ G_{[01]}(q_t, \theta; \tau_1, \tau_2) &= -n_f^+(q_t, \theta) G_{[10]}^{(0)}(q_t, \theta; \tau_1, \tau_2) + [1 - n_f^-(q_t, \theta)] G_{[01]}^{(0)}(q_t, \theta; \tau_1, \tau_2) . \end{aligned} \quad (4.1)$$

Their gluon counterparts are of a similar form,

$$\begin{aligned} D_{[10]}(k_t, \alpha; \tau_1, \tau_2) &= [1 + n_g(k_t, \alpha)] D_{[10]}^{(0)}(k_t, \alpha; \tau_1, \tau_2) + n_g(q_t, \alpha) D_{[01]}^{(0)}(q_t, \alpha; \tau_1, \tau_2) , \\ D_{[01]}(k_t, \alpha; \tau_1, \tau_2) &= n_g(k_t, \alpha) D_{[10]}^{(0)}(k_t, \alpha; \tau_1, \tau_2) + [1 + n_g(q_t, \alpha)] D_{[01]}^{(0)}(q_t, \alpha; \tau_1, \tau_2) , \end{aligned} \quad (4.2)$$

where $D_{[\alpha]}^{(0)}$ and $G_{[\alpha]}^{(0)}$ are the vacuum correlators of a given type $[\alpha]$. They are defined as vacuum expectation values of the binary products of field operators,

$$\begin{aligned} G_{[10]}^{(0)}(x_1, x_2) &= -i \langle 0 | \Psi(x_1) \bar{\Psi}(x_2) | 0 \rangle , & G_{[01]}^{(0)}(x_1, x_2) &= i \langle 0 | \bar{\Psi}(x_2) \Psi(x_1) | 0 \rangle , \\ D_{[10]lm}^{(0)}(x_1, x_2) &= -i \langle 0 | A_l(x_1) A_m(x_2) | 0 \rangle , & D_{[01]lm}^{(0)}(x_1, x_2) &= -i \langle 0 | A_m(x_2) A_l(x_1) | 0 \rangle . \end{aligned} \quad (4.3)$$

In this approximation, the field (anti-)commutators,

$$\begin{aligned} G_{[0]} &= G_{[10]} - G_{[01]} = G_{[10]}^{(0)} - G_{[01]}^{(0)} = G_{[0]}^{(0)} , \\ D_{[0]} &= D_{[10]} - D_{[01]} = D_{[10]}^{(0)} - D_{[01]}^{(0)} = D_{[0]}^{(0)} \end{aligned} \quad (4.4)$$

appear to be insensitive to the presence of the particle distribution, while their counterparts,

$$\begin{aligned} G_{[1]} &= G_{[10]} + G_{[01]} = [1 - 2n_f]G_{[1]}^{(0)}, \\ D_{[1]} &= D_{[10]} + D_{[01]} = [1 + 2n_g]D_{[1]}^{(0)}, \end{aligned} \quad (4.5)$$

include the occupation numbers which modify the original vacuum density of states. For the sake of simplicity, we take $n_f^+ = n_f^- = n_f$, which corresponds to a neutral system.

The Wightman functions (4.1) and (4.2) (or their various linear combinations $G_{[\beta]}$ and $D_{[\beta]}$) eventually appear under the integrals $d\theta$ and $d\alpha$. One must keep in mind that in order to reduce $G_{[\beta]}^{(0)}$ and $D_{[\beta]}^{(0)}$ to the standard form of the vacuum correlators, at least two shifts of the integration variables is necessary. Only after that will $G_{[\beta]}^{(0)}$ and $D_{[\beta]}^{(0)}$ explicitly depend on the boost-invariant variables η and τ_{12} . The functions $n_g(k_t, \alpha)$ and $n_f(q_t, \theta)$ are not indifferent to this shift. It may well happen that a formal shift in θ or α will drive the stationary points of the wave functions or the singularities of the field correlators outside the physical boundaries of the distributions $n_g(k_t, \alpha)$ and $n_f(q_t, \theta)$. Therefore, different representations of $G_{[1]}$ and $D_{[1]}$ must be used for the study of different subprocesses. One has to account for the reservations stemming from the derivation procedure described in Sec. 4 of paper [II]. These different representations of the quark and gluon correlators are quoted in the Appendix.

In our picture, first outlined in paper [I], the fermion vacuum mode with small transverse momentum p_t and zero rapidity is modified by its forward scattering either on gluons with high momentum k_t and rapidity α , $k_t \gg p_t$ or on quarks with high momentum q_t and rapidity θ , $q_t \gg p_t$. These hard modes are created at the earliest moment of the collision and can be treated as well formed particles by the time $\tau \sim 1/p_t$, since at that time $\tau k_t \gg 1$, and $\tau q_t \gg 1$. Therefore, they may be consistently described by the distributions,

$$n_f(q_t, \theta) \approx \frac{\mathcal{N}_f}{\pi R_\perp^2} \frac{\theta(q_t - p_*)}{q_t^2}, \quad n_g(k_t, \alpha) \approx \frac{\mathcal{N}_g}{\pi R_\perp^2} \frac{\theta(k_t - p_*)}{k_t^2}. \quad (4.6)$$

where p_* is the lower bound of the “hard” partons distribution. Both distributions (per unit area, per unit rapidity) are chosen on purely dimensional grounds, since we believe that the creation of a parton with large transverse momentum is described by perturbative QCD which has no intrinsic scale.

Currently, the normalization factors \mathcal{N}_g and \mathcal{N}_f are the only (apart from the coupling α_s) parameters of the theory. The cross section πR_\perp^2 and the full width $2Y$ of the rapidity plateau are defined by the geometry of a particular collision and the c.m.s. energy, respectively. These are irrelevant for the local screening parameters we are interested in. In the first approximation, one may try to extract them from the event-by-event measurement of the high- p_t tail of the collision products and incorporating the standard phenomenology of the fragmentation functions for the analysis.

As it was pointed out in paper [I], even in dense systems, the QCD evolution at large Q^2 is not likely to be affected by finite-density effects. Thus, one may also try to employ the known structure functions (without shadowing corrections) and the factorization scheme in order to estimate \mathcal{N}_g and \mathcal{N}_f . A most appealing opportunity to find $n_g(k_t, \alpha)$ and $n_f(q_t, \theta)$ from first principles, associating them with the known properties of hadrons and the QCD vacuum, is still very distant.

The distributions (4.6) are used below with the following informal reservations. First, the total energy of any collision is finite and k_t and q_t have (though very high, but finite) upper boundary. Eventually, this leads to the self-energy which is free from collinear singularities in the interaction of charges with the vector gauge field. Second, though the distributions (4.6) are boost-invariant, only the particles which physically affect the forward scattering must be accounted for. There is a strong correlation between the position η where the particle with large transverse momentum q_t is measured (or is interacting) and its rapidity θ . Hence, the limits of integrals $d\alpha$ and $d\theta$ over the